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Shree Datta S. S. S. K. Ltd. Charitable Trust's

SHREE DATTA POLYTECHNIC COLLEGE, DATTANAGAR.

Examination 20

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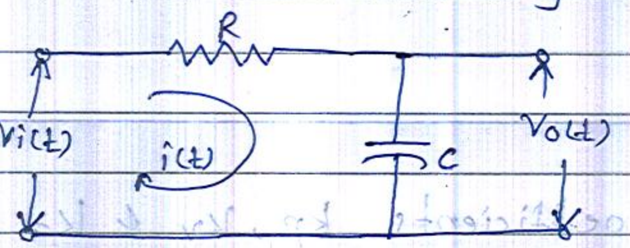
Question No.

Model Answer
Control System (12270)

Q.1 1) Find T.F. of following electrical RC circuit



Apply KVL

$\therefore V_i(t) = i(t)R + \frac{1}{C}\int i(t)dt$

Take Laplace Transform

$\therefore V_i(s) = I(s)R + \frac{1}{sC} I(s)$

Similarly, $V_o(t) = \frac{1}{C}\int i(t)dt$

Take L.T.

$\therefore V_o(s) = \frac{1}{sC} I(s)$

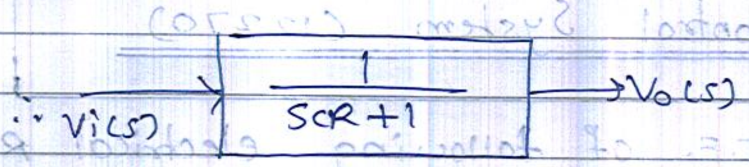
Now, $T.F. = \frac{V_o(s)}{V_i(s)}$

$$\therefore T.F. = \frac{\frac{1}{sC} I(s)}{I(s) R + \frac{1}{sC} I(s)}$$

$$= \frac{\frac{1}{sC} I(s)}{(R + \frac{1}{sC}) I(s)}$$

$$= \frac{1}{sC(R + \frac{1}{sC})}$$

$$T.F. = \frac{1}{sCR + 1}$$



2> Derive error coefficients K_p, K_v & K_a relation for Type '0' system.

→ For Type '0' System

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s) - - - -}{s^0 (1+T_a s)(1+T_b s) - - - -}$$

$$\therefore G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s) - - -}{(1+T_a s)(1+T_b s) - - -}$$

① $K_p = \lim_{s \to 0} G(s)H(s)$

$$= \lim_{s \to 0} \frac{K(1+T_1 s)(1+T_2 s) - - -}{(1+T_a s)(1+T_b s) - - -}$$

$$= \frac{K(1+0)(1+0) - -}{(1+0)(1+0) - -}$$

Pratibha

$$k_p = k$$

ii) $k_v = \lim_{s \to 0} s\,G(s)H(s)$

$$= \lim_{s \to 0} \frac{s\,k(1+T_1 s)(1+T_2 s)\cdots}{(1+T_a s)(1+T_b s)\cdots}$$

$$\therefore k_v = 0$$

iii) $k_a = \lim_{s \to 0} s^2 G(s)H(s)$

$$= \lim_{s \to 0} \frac{s^2 k(1+T_1 s)(1+T_2 s)\cdots}{(1+T_a s)(1+T_b s)\cdots}$$

$$k_a = 0$$

---

3) For given Transfer Function

$$T.F. = \frac{10(s+8)}{s(s+4)(s^2+6s+25)}$$

Find- 1) Poles
2) Zeros
3) Characteristic equation
4) Order of system

→ $$T.F. = \frac{10(s+8)}{s(s+4)(s^2+6s+25)}$$

1) Poles →

$s = 0$, $s+4 = 0$

$s = -4$

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$$s^2+6s+25=0$$

$a=1$, $b=6$, $c=25$

$$P_1P_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{36-100}}{2}$$

$$= \frac{-6 \pm \sqrt{-64}}{2}$$

$$= \frac{-6 \pm 8\sqrt{-1}}{2}$$

$$= -3 \pm 4\sqrt{-1}$$

$$\therefore P_1 = -3+4j \quad , \quad P_2 = -3-4j$$

2) Zeros →

$$s+8=0$$

$$s=-8$$

3) Characteristic Equation →

$$s(s+4)(s^2+6s+25)=0$$

$$(s^2+4s)(s^2+6s+25)=0$$

$$s^4+6s^3+25s^2+4s^3+24s^2+100s=0$$

$$s^4+10s^3+49s^2+100s=0$$

4) Order of System →

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Question No.

4) A system has

$$G(s)H(s) = \frac{k(s+13)}{s(s+3)(s+7)}$$

where k is positive, Determine the range of k values for system stability.

→ Characteristic eqn is given by,

$1 + G(s)H(s) = 0$

$\therefore 1 + \frac{k(s+13)}{s(s+3)(s+7)} = 0$

$\Rightarrow s(s+3)(s+7) + k(s+13) = 0$

$\therefore (s^2+3s)(s+7) + ks + 13k = 0$

$\therefore s^3 + 7s^2 + 3s^2 + 21s + ks + 13k = 0$

$\therefore s^3 + 10s^2 + (21+k)s + 13k = 0$

Now,

Routh Array is,

s^3	1	$21+k$	0
s^2	10	$13k$	
s^1	$\frac{210+10k-13k}{10}$	0	
s^0	$13k$		

where system is stable

$\therefore \quad k > 0$

$\therefore \quad \frac{210+10k-13k}{10} > 0$

$\therefore \quad 210+10k-13k > 0$

$\therefore \quad 210-3k > 0$

$\therefore \quad 210 > 3k$

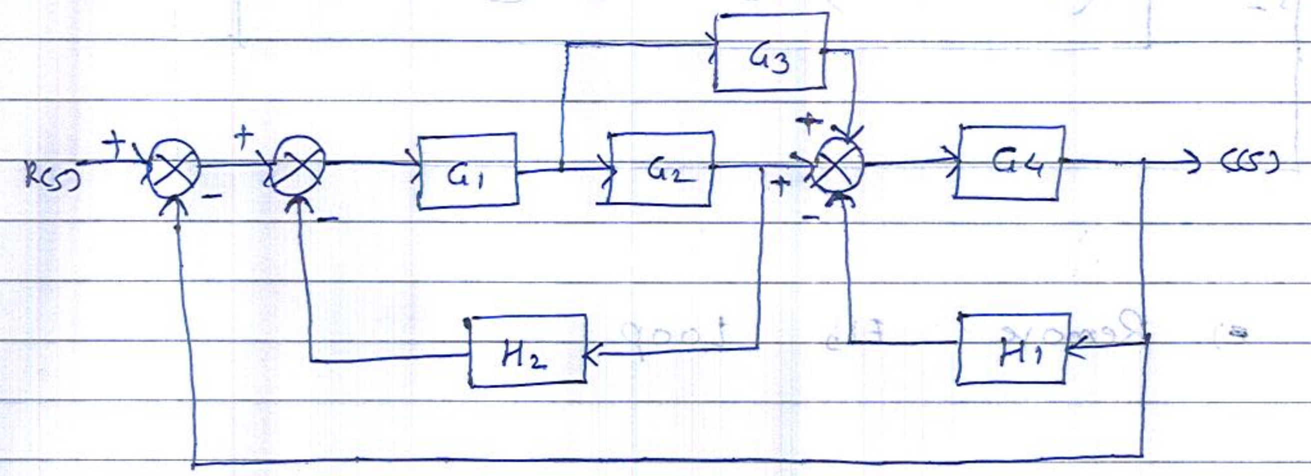
$\therefore \quad 70 > k$

$\boxed{\therefore \quad 70 > k > 0}$

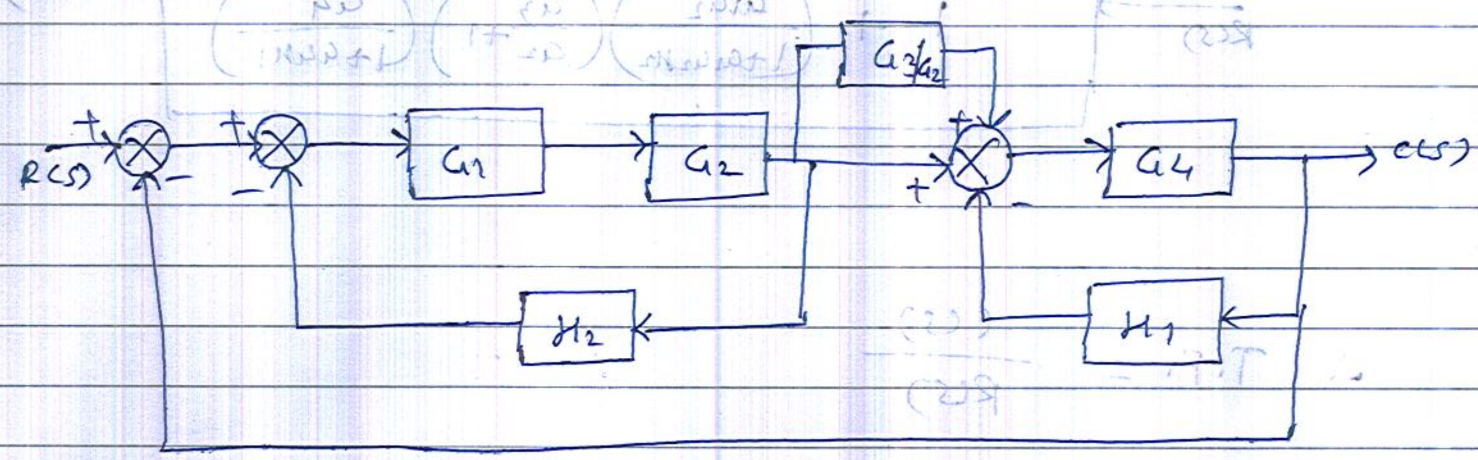
Is the range of value of k.

Pratibha

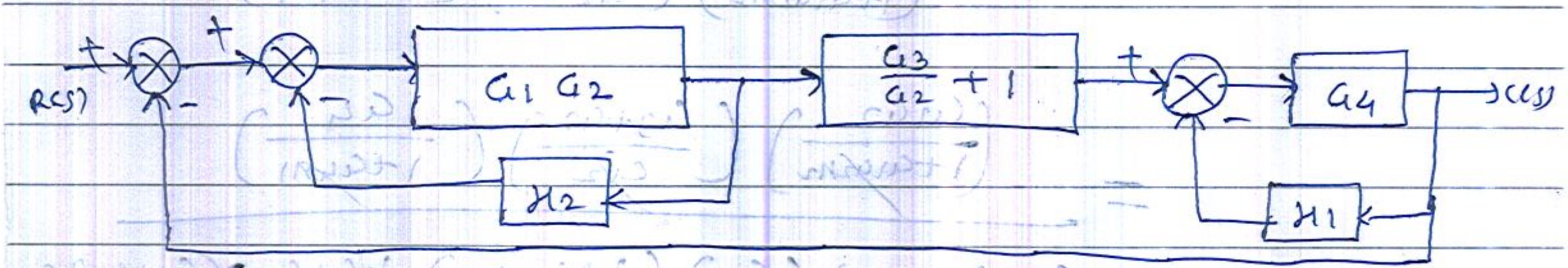
Q. 2) 1) Reduce the given complex block diagram in its simple form using reduction rules.
Find overall T.F. C(s)/R(s)



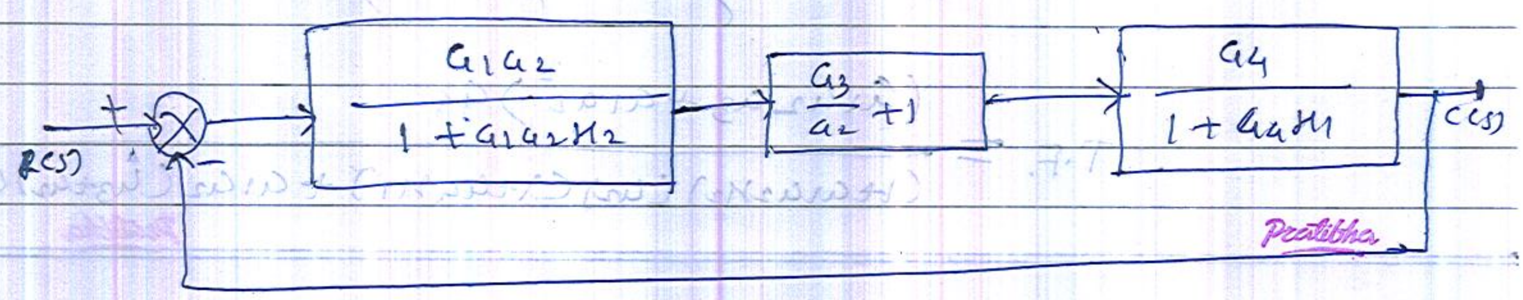
→ a) Shift take-off point after G2 block



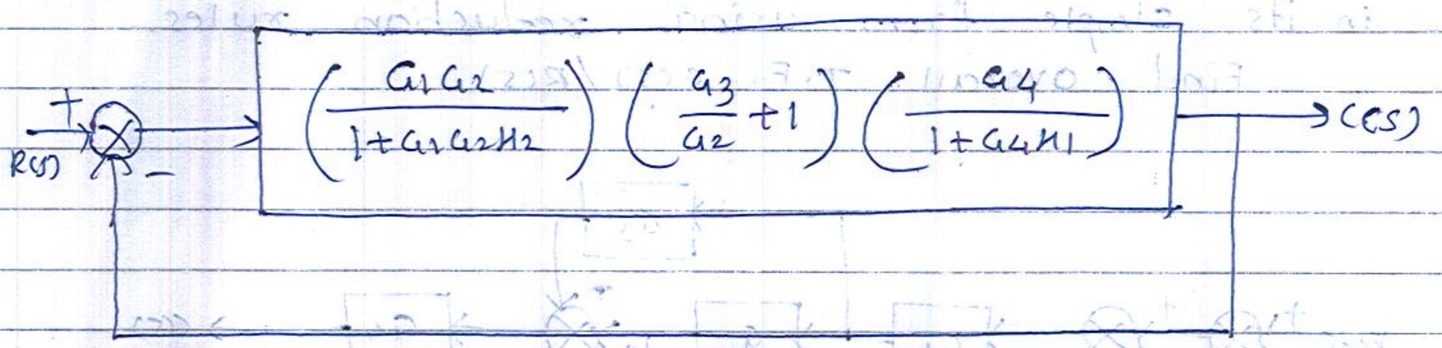
b) Use series combination and Parallel combn.



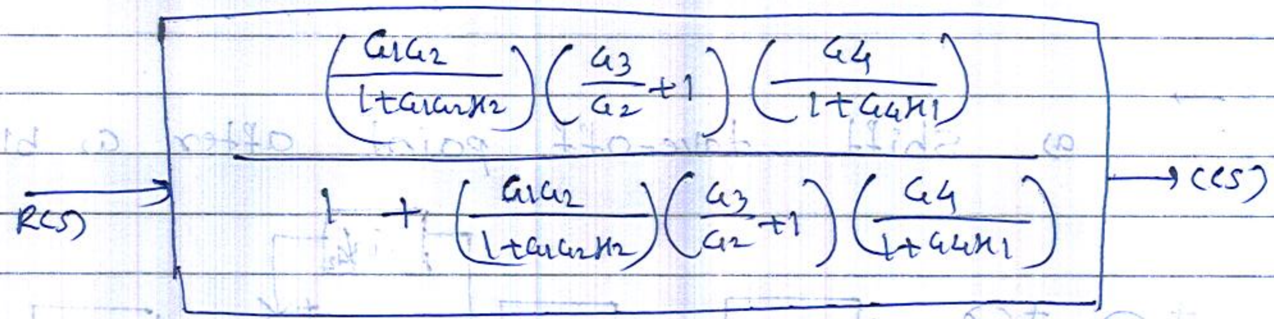
c) Remove F/b Loop



d) Use Series Combination



e) Remove F/b Loop



$$\therefore T.F. = \frac{C(s)}{R(s)}$$

$$= \frac{\left(\frac{G_1G_2}{1+G_1G_2H_2}\right)\left(\frac{G_3}{G_2}+1\right)\left(\frac{G_4}{1+G_4H_1}\right)}{1+\left(\frac{G_1G_2}{1+G_1G_2H_2}\right)\left(\frac{G_3}{G_2}+1\right)\left(\frac{G_4}{1+G_4H_1}\right)}$$

$$= \frac{\left(\frac{G_1G_2}{1+G_1G_2H_2}\right)\left(\frac{G_3+G_2}{G_2}\right)\left(\frac{G_4}{1+G_4H_1}\right)}{\frac{(1+G_1G_2H_2)(G_2)(1+G_4H_1)+(G_1G_2)(G_3+G_2)G_4}{(1+G_1G_2H_2)(G_2)(1+G_4H_1)}}$$

$$\therefore T.F. = \frac{(G_1G_2G_3+G_1G_2^2)G_4}{(1+G_1G_2H_2)(G_2)(1+G_4H_1)+G_1G_2(G_3+G_2)G_4}$$

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Question No.

2) For the system with characteristic equation

$$s^4 + 6s^3 + 26s^2 + 56s + 80 = 0$$

Determine stability.

→ Routh Array is given by,

s^4	1	26	80
s^3	6	56	
s^2	16.67	80	
s^1	27.20	0	
s^0	80		

As, there is no sign changes in the 1st column of array.

So, the system is stable system.

3) For the given differential equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y(t) = 8x(t)$$

Where, y = O/P, x = I/P

Find - i) Settling time

ii) Rise time

iii) Peak time

iv) Peak overshoot

→

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y(t) = 8x(t)$$

Take L.T.

$$\therefore s^2Y(s) + 4sY(s) + 8Y(s) = 8X(s)$$

$$\therefore (s^2 + 4s + 8)Y(s) = 8X(s)$$

$$\therefore \frac{Y(s)}{X(s)} = \frac{8}{s^2+4s+8}$$

Compairing with $\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$\therefore \omega_n^2 = 8$

$\therefore \omega_n = 2.83$ rad/sec

$2\xi\omega_n = 4$

$\xi\omega_n = 2$

$\xi = \frac{2}{\omega_n}$

$\therefore \xi = 0.7$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 2.83\sqrt{1-0.7^2}$$

$\therefore \omega_d = 2.002$ rad/sec

Pratibha

i) Settling time (t_s) →

$$t_s = \frac{4}{\xi \omega_n}$$

$$= \frac{4}{0.7 \times 2.83}$$

$$\therefore t_s = 2 \text{ sec}$$

ii) Rise time (t_r) →

$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{1-0.7^2}}{0.7}\right)$$

$$\therefore \theta = 45.03^\circ = \frac{\pi}{4} \text{ rad}$$

$$\therefore t_r = \frac{\pi - \frac{\pi}{4}}{2.002}$$

$$\therefore t_r = 1.17 \text{ sec}$$

iii) Peak time (t_p) →

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{2.002}$$

$$\therefore t_p = 1.57 \text{ sec}$$

iv) Peak Overshoot (M_p) →

$$\% M_p = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} \times 100$$

$$= e^{-\pi \times 0.17 / \sqrt{1-0.17^2}} \times 100$$

$$\therefore \% M_p = 4.33\%$$

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Question No.

Q.3 i) A second order system is given by,

$$\frac{C(s)}{R(s)} = \frac{25}{s^2+6s+25}$$

Find i) Damping ratio and Natural Frequency
ii) Peak time
iii) Peak Overshoot
iv) Settling time.

→ $$\frac{C(s)}{R(s)} = \frac{25}{s^2+6s+25}$$

Compairing with $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$\therefore \omega_n^2 = 25$

$\omega_n = 5$ rad/sec

$2\xi\omega_n = 6$

$\xi\omega_n = 3$

$\xi = \frac{3}{5}$

$\xi = 0.6$

b) $\omega_d = \omega_n\sqrt{1-\xi^2}$

$= 5\sqrt{1-0.6^2}$

$\therefore \omega_d = 4$ rad/sec

Pratibha

2) Rise time $t_r \rightarrow$

$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{1-0.6^2}}{0.6}\right)$$

$$\therefore \theta = 0.9272\,rad$$

$$\therefore t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - 0.9272}{4}$$

$$\therefore t_r = 0.55\,sec$$

iii) Peak Overshoot (%Mp) →

$$\%M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = e^{-\pi 0.6/\sqrt{1-0.6^2}} \times 100$$

$$\%M_p = 9.48\,\%$$

iv) Settling time (t_s) →

$$t_s = \frac{4}{\xi\omega_n}$$

$$= \frac{4}{0.6 \times 5}$$

$$t_s = 1.33\,sec$$

2) Consider 5^{th} order system with characteristic equation given by,

$$s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$$

Determine stability of system using Routh Criterion.

→ Routh Array is,

s^5	1	2	3
s^4	1	2	5
s^3	$0 = \epsilon_1$	-2	
s^2	$\frac{2\epsilon_1 + 2}{\epsilon_1}$	5	
s^1	$\frac{-2\left(\frac{2\epsilon_1+2}{\epsilon_1}\right) - 5\epsilon}{\frac{2\epsilon_1+2}{\epsilon_1}}$	0	
s^0	5		

As $\frac{2\epsilon_1 + 2}{\epsilon_1}$ is +ve

So $\frac{-2\frac{(2\epsilon_1+2)}{\epsilon_1} - 5\epsilon_1}{\frac{2\epsilon_1+2}{\epsilon_1}}$

$$= \frac{\frac{-2(2\epsilon_1+2) - 5\epsilon_1^2}{\epsilon_1}}{\frac{2\epsilon_1+2}{\epsilon_1}}$$

$$= \frac{-2(0+2) - 0}{0+2} = \frac{-4}{2} = -2$$

So there is two sign changes in 1st column of Array. So the system is <u>Unstable system</u>.

3) A unity F/b system has G(s)

$$G(s) = \frac{10(s+1)}{s^2(s+2)(s+10)}$$

Find i) Error coefficients K_p, K_v and K_a
ii) Type of system
iii) Steady state error for input
$r(t) = 1 + 4t + \frac{t^2}{2}$

→

$$G(s) = \frac{10(s+1)}{s^2(s+2)(s+10)}$$

$$H(s) = 1$$

$$\therefore G(s)H(s) = \frac{10(s+1)}{s^2(s+2)(s+10)}$$

i)

$$K_p = \lim_{s \to 0} \frac{10(s+1)}{s^2(s+2)(s+10)}$$

$$= \frac{10(0+1)}{0(0+2)(0+10)}$$

$$= \frac{10}{0}$$

$$\boxed{\therefore K_p = \infty}$$

$$K_a = \lim_{s \to 0} s\, G(s)H(s)$$

$$= \lim_{s \to 0} s \frac{10(s+1)}{s^2(s+2)(s+10)}$$

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$$k_v = \frac{10(0+1)}{0(0+2)(0+10)}$$

$$= \frac{10}{0}$$

$$\therefore k_v = \infty$$

$$k_a = \lim_{s \to 0} s^2 G(s)H(s)$$

$$= \lim_{s \to 0} \cancel{s^2} \frac{10(s+1)}{\cancel{s^2}(s+2)(s+10)}$$

$$= \frac{10(0+1)}{(0+2)(0+10)}$$

$$= \frac{\cancel{10}1}{\cancel{20}2}$$

$$= \frac{1}{2}$$

$$\therefore k_a = 0.5$$

iii) Type of System = 2

iii)

$$r(t) = 1 + 4t + \frac{t^2}{2}$$

1	$4t$	$\frac{t^2}{2}$
step i/p	Ramp i/p	Parabolic i/p
A=1	A=4	A=1

$$\therefore e_{ss_1} = \frac{A}{1+K_p}$$

$$= \frac{1}{1+\infty}$$

$$= \frac{1}{\infty}$$

$$\boxed{\therefore e_{ss_1} = 0}$$

$$e_{ss_2} = \frac{A}{K_v}$$

$$= \frac{4}{\infty}$$

$$\boxed{e_{ss_2} = 0}$$

$$e_{ss_3} = \frac{A}{K_a}$$

$$= \frac{1}{0.5}$$

$$\boxed{e_{ss_3} = 2}$$

$\therefore$ Steady state Error is given by,

$$e_{ss} = e_{ss_1} + e_{ss_2} + e_{ss_3}$$

$$= 0 + 0 + 2$$

$$\boxed{\therefore e_{ss} = 2}$$

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